OCR
RECOGNISING ACHIEVEMENT

Thursday 31 May 2012 – Morning

AS GCE MATHEMATICS

4728 Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4728
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

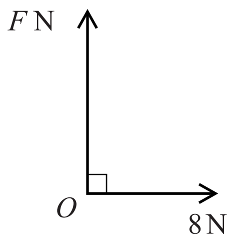
This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1



Two perpendicular forces of magnitudes F N and 8 N act at a point O (see diagram). Their resultant has magnitude 17 N.

(i) Calculate F and find the angle which the resultant makes with the 8 N force. **[4]**

A third force of magnitude E N, acting in the same plane as the two original forces, is now applied at the point O. The three forces of magnitudes E N, F N and 8 N are in equilibrium.

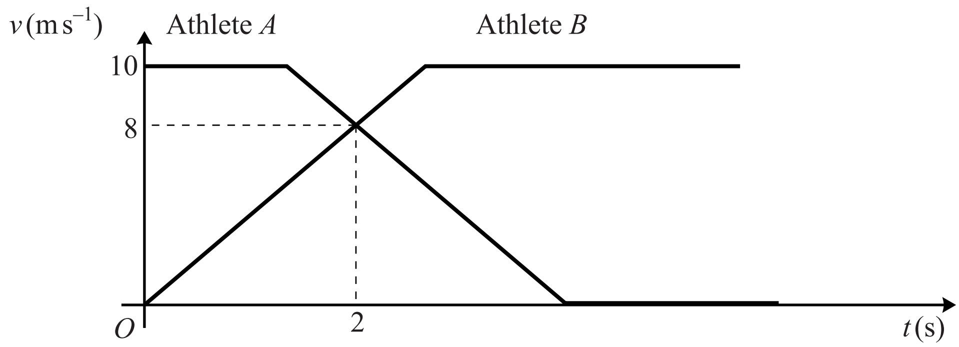
(ii) State the value of E and the angle between the directions of the E N and 8 N forces. **[2]**

2 A particle is projected vertically upwards with speed $7\,\mathrm{m\,s^{-1}}$ from a point on the ground.

(i) Find the speed of the particle and its distance above the ground 0.4 s after projection. **[4]**

(ii) Find the total distance travelled by the particle in the first 0.9 s after projection. **[4]**

3



not to scale

The diagram shows the (t, v) graphs for two athletes, A and B, who run in the same direction in the same straight line while they exchange the baton in a relay race. A runs with constant velocity $10\,\mathrm{m\,s^{-1}}$ until he decelerates at $5\,\mathrm{m\,s^{-2}}$ and subsequently comes to rest. B has constant acceleration from rest until reaching his constant speed of $10\,\mathrm{m\,s^{-1}}$. The baton is exchanged 2 s after B starts running, when both athletes have speed $8\,\mathrm{m\,s^{-1}}$ and B is 1 m ahead of A.

(i) Find the value of t at which A starts to decelerate. **[2]**

(ii) Calculate the distance between A and B at the instant when B starts to run. **[5]**

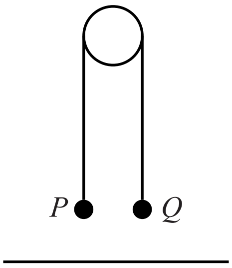
4 A block B of weight 28 N is pulled at constant speed across a rough horizontal surface by a force of magnitude 14 N inclined at 30° above the horizontal.

(i) Show that the coefficient of friction between the block and the surface is 0.577, correct to 3 significant figures. **[4]**

The 14 N force is suddenly removed, and the block decelerates, coming to rest after travelling a further 3.2 m.

(ii) Calculate the speed of the block at the instant the 14 N force was removed. **[6]**

5



Particles P and Q, of masses 0.4 kg and m kg respectively, are joined by a light inextensible string which passes over a smooth pulley. The particles are released from rest at the same height above a horizontal surface; the string is taut and the portions of the string not in contact with the pulley are vertical (see diagram). Q begins to descend with acceleration $2.45\ \text{m s}^{-2}$ and reaches the surface 0.3 s after being released. Subsequently, Q remains at rest and P never reaches the pulley.

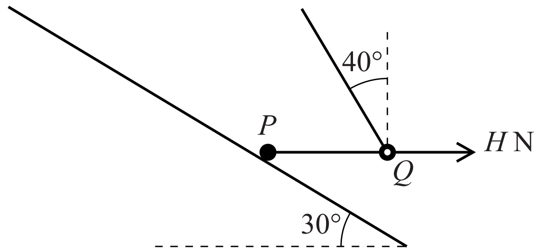
(i) Calculate the tension in the string while Q is in motion. **[3]**

(ii) Calculate the momentum lost by Q when it reaches the surface. **[5]**

(iii) Calculate the greatest height of P above the surface. **[5]**

[Questions 6 and 7 are printed overleaf.]

6



A particle P lies on a slope inclined at 30° to the horizontal. P is attached to one end of a taut light inextensible string which passes through a small smooth ring Q of mass m kg. The portion PQ of the string is horizontal and the other portion of the string is inclined at 40° to the vertical. A horizontal force of magnitude H N, acting away from P, is applied to Q (see diagram). The tension in the string is 6.4 N, and the string is in the vertical plane containing the line of greatest slope on which P lies. Both P and Q are in equilibrium.

(i) Calculate m. **[2]**

(ii) Calculate H. **[2]**

(iii) Given that the weight of P is 32 N, and that P is in limiting equilibrium, show that the coefficient of friction between P and the slope is 0.879, correct to 3 significant figures. **[6]**

Q and the string are now removed.

(iv) Determine whether P remains in equilibrium. **[3]**

7

The diagram shows two particles P and Q, of masses 0.2 kg and 0.3 kg respectively, which move on a horizontal surface in the same direction along a straight line. A stationary particle R of mass 1.5 kg also lies on this line. P and Q collide and coalesce to form a combined particle C. Immediately before this collision P has velocity $4\,\text{m s}^{-1}$ and Q has velocity $2.5\,\text{m s}^{-1}$.

(i) Calculate the velocity of C immediately after this collision. **[3]**

At time t s after this collision the velocity $v\,\text{m s}^{-1}$ of C is given by $v = V_0 - 3t^2$ for $0 < t \leqslant 0.3$. C strikes R when $t = 0.3$.

(ii) (a) State the value of V_0. **[1]**

(b) Calculate the distance C moves before it strikes R. **[4]**

(c) Find the acceleration of C immediately before it strikes R. **[3]**

Immediately after C strikes R, the particles have equal speeds but move in opposite directions.

(iii) Find the speed of C immediately after it strikes R. **[4]**

Question			Answer	Marks	Guidance
1	(i)		$F^2 = 17^2 - 8^2$ $F = 15$ cosα = 8/17 α = 61.9°	M1 A1 M1 A1 **[4]**	$F^2 = 17^2$ +/– 8^2 Exact accept 15.0 Correct method for angle between 8 N and 17 N forces Accept 62° from correct work
1	(ii)		$E = 17$ Angle = 118(.1)° OR 242° (241.9°)	B1 B1 FT **[2]**	Exact 180 – cv(α(**i**)) OR 180 +cv(a(**i**)) Must be 3sf or better
2	(i)		$v = 7 - 0.4 \times 9.8$ $v = 3.08$ ms^{-1} $s = 7 \times 0.4 - 9.8 \times 0.4^2/2$ $s = 2.016$ m OR $3.08^2 = 7^2 - 2 \times 9.8s$ $s = 2.016$ m OR $v^2 = 7^2 - 2 \times 9.8 \times 2.016$ $v = 3.08$ ms^{-1}	M1 A1 M1 A1 **[4]** M1 A1 M1 A1	v = 7 +/– 0.4g Exact, or correct to 3sf from g=9.81(3.076) or 10 (3) s = 7 × 0.4 +/– $g0.4^2/2$ Exact but accept 2.02. g=9.81 (2.0152) or g=10 (2) $(\text{cv}(v))^2 = 7^2$ +/– $2gs$ Exact but accept 2.02. g=9.81 (2.0152) or g=10 (2) $v^2 = 7^2$ +/– $2g(\text{cv}(s))$ Exact or correct to 3sf. Accept v=3.07 from s=2.02. From g=9.81(3.076 or 3.06 from s=2.02) or 10 (3)
2	(ii)		$H = \pm 7^2/(2 \times 9.8)$ (= ±2.5) $S = \pm (7 \times 0.9 - ½ \times 9.8 \times 0.9^2)$ (= ±2.331) $D = 2.5 + (2.5 - 2.331)$ $D = 2.669$ m OR (Using t_U = 7/9.8=0.7143, t_D = 0.9–0.7143= 0.1857 s) $H = \pm (7\text{x}0.7143 - 9.8\text{x}0.7143^2/2)$ (= ±2.5) $s_D = \pm 9.8\text{x}0.1857^2/2$ (= ±0.169) $D = 2.5 + 0.169$ $D = 2.669$ m	B1 B1 M1 A1 **[4]** B1 B1 M1 A1	Greatest Height, g=9.81 (2.497 accept 2.5) g=10 (2.45) Height when t = 0.9, g=9.81 (2.32695) g=10 (2.25) 2 × greatest height – S(0.9) Exact but accept 2.67, g=9.81 (2.66705) g=10 (2.65) "OR" method uses distance from greatest height. OR ±9.8x0.7143^2/2. Gains B1 for H as above Equivalent to B1 for S as above Greatest height + Descent distance << H Exact but accept 2.67, g=9.81 (2.66705) g=10 (2.65)

Question			Answer	Marks	Guidance
3	(i)		$(10-8)/5 = T_{dec}$ OR $8 = 10 - 5T_{dec}$ t $(= 2 - 0.4) = 1.6$	M1 A1 **[2]**	Attempt to find $T_{dec} = \pm 0.4 = \pm 2/5$ Exact. Accept 1 3/5, not 8/5, www
3	(ii)		$S_B = ½ \times 8 \times 2$ $S_A = 10 \times 1.6 + ½ \times (10+8) \times 0.4$ OR $S_A = 10 \times 2 - ½ \times (2-1.6) \times (10-8)$ $S_A = 19.6$ $AB = 19.6 - 8 + 1$ $AB = 12.6$ m	B1 M1 A1 M1 A1 **[5]**	$S_B = 8$ Using area under graph is distance (at least two parts) Complete method for S_A run in the first 2s, using cv(t) Accept as 16+3.6 or 20-0.40, from t =1.6 (however obtained) $AB = +/-(S_A - S_B +/- 1)$ Exact Or $AB = -12.6$ m
4	(i)		$Fr = 14\cos 30$ $R = 28 - 14\sin 30$ $(14\cos 30) = \mu\ (28 - 14\sin 30)$ $\mu = 0.577$ AG	B1 B1 M1 A1 **[4]**	12.1(24..) 21 12.1(24..)/21. Allow |component of 14|/|cv(R)| for M1 0.577(35..)
4	(ii)		Mass = 28/g $Fr = 0.577 \times 28$ $(28/9.8)a = \pm 0.577 \times 28$ $a = \pm 5.66$ from exact μ, $a = \pm 5.65$ from μ = 0.577 $0 = u^2 - 2 \times 5.66 \times 3.2$ $u = 6.02$ m s^{-1}	B1 B1 M1 A1 M1 A1 **[6]**	2.857.. Award here if seen in (i) <u>and</u> used in (ii) 16.156 or 0.57735.. x 28 = 16.1658.. Award also for cv(m), m = 28. Must be only one force (friction), allow Fr(i). g=10 (±5.77) Valid signs with cv(5.66) Accept any answer rounding to 6.0 (inc 6.0, not 6) or 6.1 from g=10

Question			Answer	Marks	Guidance
5	(i)			M1	N2L on P , two vertical forces, accept with 0.4x2.45g
			$T - 0.4g = 0.4 \times 2.45$	A1	Correct terms and signs
			$T = 4.9$ N	A1	Exact, g=9.81 (4.904, accept 4.9) g=10 (4.98, not 5.0)
				[3]	
5	(ii)		$mg - T = \pm 2.45m$	M1	Correct terms (possible incorrect signs), and use of cv(T(i))
			$m = 2/3$ kg	A1 FT	FT cv(T(i))/7.35, g=9.81 (FT cv(T(i))/7.351 = 0.667)g=10 (FT cv(T(i))/7.55 = 0.6596 = 0.66) This may be seen in (i). The M1A1 pair of marks may be awarded only in part (ii) when the candidate uses the value of m which was found in (i).
			$v = 2.45 \times 0.3$ (= 0.735)	B1	Must be positive
			Momentum = (2/3) × (2.45 × 0.3)	M1	Accept ±. cv(m) x cv(v)
			Momentum loss = 0.49 kgms^{-1}	A1	Exact,. but accept any value which rounds to ±0.490. g=9.81 (0.49) g=10 (0.4848=0.485, not 0.48)
				[5]	
5	(iii)		$S = 2.45 \times 0.3^2/2$	M1	Distance while Q descends. Watch for $s = vt\text{-}at^2/2$. If v=0, M0A0
			$S = \pm\ 0.11(025)$	A1	
			OR		
			$S = (0 + 0.735) \times 0.3 / 2$		M1 Using landing speed from (ii)
			$S = \pm\ 0.11(025)$		A1
			$0 = (2.45 \times 0.3)^2 \pm 2 \times 9.8s$	M1	Distance P ascends while Q at rest, must use g
			$s = \pm 0.027(56..)$	A1	May be implied, g=9.81 (0.02753) g=10 (0.0270)
			OR (using $t_A = 0.735/9.8 = 0.075$)		Calculating ascend time after string goes slack
			$s = 0.735 \times 0.075 - 9.8 \times 0.075^2 / 2$		M1 Using candidate's values of speed and t_A to find $\pm s$
			$s = \pm\ 0.027(56..)$		A1 May be implied
			Distance = 0.248 m	A1 FT	2 × |cv(S)| + |cv(s)|. Accept 0.25. g=9.81 (0.248) g=10 (0.247511..)
				[5]	

Question			Answer	Marks	Guidance
6	(i)		$mg = 6.4\cos 40$ $m = 0.5(00)$	M1 A1 **[2]**	One cmpt of 6.4 N force (allow 6.4 x sin/cos 40 or 50), mg not resolved Accept 0.5, g=9.81 (0.49976..=0.5) g=10 (0.49026.. = 0.49)
6	(ii)		$H = 6.4 + 6.4\sin 40$ OR $2 \times 6.4\cos 25 = 0.5g\cos 65 + H\cos 25$ $H = 10.5$	 M1 A1 **[2]**	Resolves horizontally, all necessary terms (allow e.g. 6.4 ± 6.4cos40) Resolves parallel to bisector of strings, inc cmpt weight Accept 11
6	(iii)		$R = 32\cos 30 – 6.4\sin 30$ $R = 24.5$ $Fr = 32\sin 30 + 6.4\cos 30$ $Fr = 21.5$ $\mu = (32\sin 30 + 6.4\cos 30)/(32\cos 30 - 6.4\sin 30)$ $\mu = 0.879$ AG	M1 A1 M1 A1 M1 A1 **[6]**	Difference of Wt cmpt and Tension (not H) cmpt May be implied Sum of Wt cmpt and Tension (not H) cmpt May be implied **Either** Fr **or** R obtained from 2 term numerical expressions, in $\|Fr\| = \mu\|R\|$
6	(iv)		$F_{max} = 0.879 \times 32\cos 30$ (= 24.4 N) Wt cmpt down slope = 32sin30 (= 16 N) Remains in eqbm OR $\pm ma = 32\sin 30 - 0.879 \times 32\cos 30$ Finds acceleration Remains in eqbm OR angle of friction = $\tan^{-1}0.879 = 41°$ Slope is 30° Remains in eqbm	B1* D*M1 A1 **[3]** B1* D*M1 A1 B1* D*M1 A1	May be described simply as F or friction Finding Wt component down slope and comparing with friction Needs Wt cmpt = 16 < F_{max} For friction calculation Sets up and solves N2L for a Needs a clearly in direction of friction (impossible) Must be explicit Values of angle of friction and slope stated in 6(iv)

Question			Answer	Marks	Guidance
7	(i)		Before mom = 0.2 × 4 + 0.3 × 2.5 0.2 × 4 + 0.3 × 2.5 = (0.2 + 0.3)v v = 3.1 ms^{-1}	B1 M1 A1 **[3]**	Accept with g Accept with g Exact. Award if g used and cancelled.
7	(ii)	(a)	$V_0 = 3.1$	B1 FT **[1]**	FT cv(v(i))
7	(ii)	(b)	$s = \int 3.1 - 3t^2 \mathrm{d}t$ $s = 3.1t - 3t^3/3$ (+c) CR = $[3.1t - t^3]_0^{0.3}$ CR = 0.903 m	M1* A1 FT D*M1 A1 **[4]**	Uses integration of velocity(t) FT cv(v(i)) or cv(V_0(iia)) Uses their s(0.3). Award if +c never shown or assumed = 0 Ans not given, so explicit substitution not needed. Allow 0.90, not 0.9
7	(ii)	(c)	$a = d(V_0 - 3t^2)/dt$ $a = -6 \times 0.3$ $a = -1.8$ ms^{-2}	M1* D*M1 A1 **[3]**	Uses differentiation of v Substitutes t = 0.3 (no other value acceptable) Exact. Must be negative (accept deceleration is -1.8). Award if V_0 wrong but not if V_0 omitted.
	(iii)		Mom C = (0.2 + 0.3)(3.1 – 3 × 0.3^2) Conservation of momentum used, no g (0.2 + 0.3)(3.1 – 3 × 0.3^2) = 1.5v – 0.5v v = 1.415 ms^{-1}	B1 M1 A1FT A1 **[4]**	1.415 Before momentum must be numerical, after momentum needs two terms in v (accept 2v or v) FT cv(before momentum) Exact. Accept 1.41 or 1.42.